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Animating Horse Gaits and Transitions

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Abstract

Animations of horses are commonly used for entertainment purposes. A realistic animated model must move with a gait appropriate to its velocity. We present a kinematic animation system in which a horse model moves using gaits and transitions based on predictions from Dynamic Similarity theory. A Genetic Programming technique is used to evolve gait motion with dynamically adjustable limb extent. The system is controlled in real-time using a MIDI controller system based around the model's Froude number. We were successful in producing high quality animations of the horse's natural gaits and transitions.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

1. Introduction

Animations of quadruped animals are frequently present in films, on television and in computer games. As the viewer is often familiar with a portrayed animal, unrealistic motions can be obvious. If the goal is to depict an animal's motion realistically, inconsistencies with nature reduce an animation's effectiveness. One of the common mistakes observed is when an animated animal moves with an incorrect gait.

A gait is the pattern in which an animal moves its limbs and could be considered incorrect if the leg motion is perceived to be inconsistent with the velocity at which the model as a whole is moving. For example, a model translating at a slow speed whilst its limbs move in a gallop pattern is visually incorrect and physically implausible. Similarly, a model translating at a fast rate whilst using the limb pattern of a walk is equally incorrect. If an animation is to be realistic, the model should move with a gait appropriate to its velocity and be able to transition smoothly between gaits.

Gait transitions occur as an animal changes its velocity. The form of these transitions is often idiosyncratic to a particular animal and can be inconsistent. As such there are no set transition patterns. It is also difficult to assess how a transition occurs without specialist equipment. This lack of information motivates the need for a method to determine an appropriate transition pattern between arbitrary gaits.

We present a system in which a horse model is kinematically animated to move with gaits appropriate to its velocity and transition between adjacent gaits. Gait and transition properties are predicted using Dynamic Similarity theory and the dimensionless Froude number. We describe two data representations for use with cyclic (gait) and acyclic (transition) motions respectively. For realism, intra-gait increases in velocity cause increases in limb extent as well as stride frequency. To dynamically adjust motion data for the current velocity, we evolve gait adjusters using an Evolutionary Computation technique. We also introduce a novel animation control system utilising a MIDI (Musical Instrument Digital Interface) controller and the model's Froude number. We conclude with analysis of the resultant animations.

2. Related work

Quadrupedal animation is a well-studied topic [SRH*09]. The creation of realistic quadrupedal animations is reliant on a solid knowledge of animal movement. The photographs of Muybridge [Muy85] provide animators with an extensive visual record of animal locomotion. Publications such as [BC01, Har93] provide a wealth of knowledge and data regarding equine locomotion whilst the nature of gait transitions has been explored in many disciplines [Mar74, LS02].

The similarity between the gaits of animals across a range of species have been explored in Dynamic Similarity theory [AJ83]. This theory and the Froude number [VO05] upon which it is based allows one to predict the gait characteristics of an animal at a given velocity. These predictions are used to develop and verify horse gait systems in [HM01].

Dynamic Similarity predictions are used as the fitness function in a Grammatical Evolution (GE) approach to gait optimisation in [MOC09]. GE is a grammar-based form of Genetic Programming [OR03]. Solutions to a problem are evolved generation by generation based on principles adapted from molecular biology and the concept of phenotypic fitness. This technique successfully optimises gaits for use with a physics-based horse model.

3. Gaits and transitions

A horse has four natural gaits. In order of increasing speed, they are the walk, trot, canter and gallop. Transitions occur between these gaits at predictable velocities, generally to minimise energy cost.

The natural gaits of the horse are presented in Figure 1 with the phase differences between the limbs, from [BC01], given as a percentage of a full cycle. The walk and trot are symmetrical gaits meaning a left and right pair of limbs move alternately to each other. For asymmetrical gaits such as the canter and gallop, pairs of limbs move together. This symmetry is visible in the stick diagrams which plot the footfall sequences for each gait.

In general, a particular gait is efficient over a range of velocities. The animal can increase or decrease its velocity within that range without transitioning to a new gait. The animal changes its velocity within a gait by adjusting the length of its step (limb extent) as well as its stride frequency. An increase in a limb's extent increases velocity, even if stride frequency remains the same. In order of increasing limb extent, gaits are divided into collected, medium or extended types. In nature, animals often transition up or down a gait rather than adopt a highly extended or collected gait respectively.

As a horse is transitioning between gaits, it either increases or decreases its limbs' rate of movement to achieve a target phase difference. The pattern of change and time taken to complete a transition varies hugely depending on factors such as current gait and velocity, target gait and velocity, terrain, breed or habit. This inconsistency means that transitions take a varying number of strides to complete. The phase differences however are consistent among quadrupeds for the natural gaits and the velocity at which these transitions occur is also found to be relatively equal.

3.1. Dynamic Similarity predictions

Dynamic Similarity theory allows one to predict the gait characteristics a quadrupedal mammal will exhibit when travelling at a particular velocity. It is based on a dimensionless value called the Froude number which relates velocity, acceleration due to gravity and height of the hip from the ground. The predictions depend on the availability of gait data measured from other quadrupedal mammals of differing species, such as that found in [AJ83].

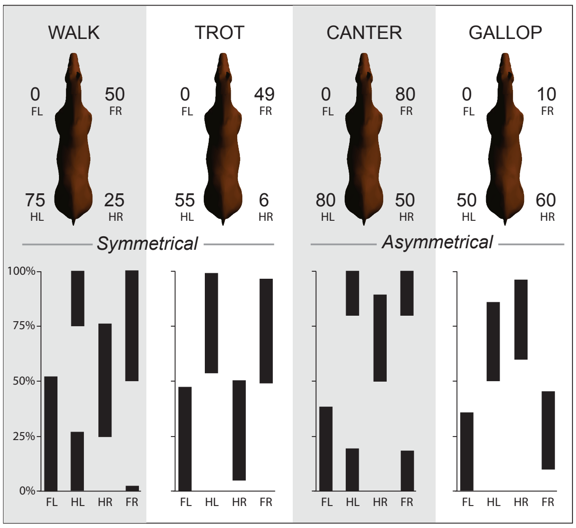

Figure 1: *Top: typical timing of footfalls as a % of a full gait cycle, for the natural gaits (FL: fore-left, FR: fore-right, HL: hind-left, HR: hind-right). Bottom: hoof contact with the ground (thick black bars indicate contact).*

The theory is based on the observation that animals moving at equal Froude numbers have similar gaits. The Froude number is calculated by Equation 1.

$$Fr = v^2/gh \tag{1}$$

where Fr is the Froude number, v is velocity, g is gravity and h is height of the hip from the ground at stance. Experiments show that animals travelling at the same Froude number have similar limb phase relationships, relative stride lengths and duty factors. Stride length is the distance travelled between an initial ground contact of a foot, and the next grounding of the same foot. Duty factor is the percentage of a gait cycle that a limb is in contact with the ground for.

Table 1: *Gaits and corresponding Froude numbers*

Gait	Walk	Trot	Canter	Gallop
Froude	0.0-1.5	1.51-2.5	2.51-3.5	3.51-4.5
Symmetry	Sym.	Sym.	Asym.	Asym.

Figure 2 contains a plot of velocity versus Froude number for a specific horse, annotated with the predicted gaits shown in Table 1. This table presents typical Froude number ranges for the natural gaits, taken from [AJ83].

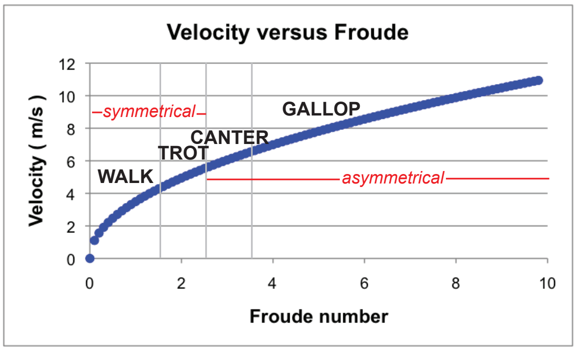

Figure 2: *Velocity versus Froude number as predicted by the Froude equation (hip-height: 1.24524). The predicted gaits, transition points and gait symmetries are also displayed.*

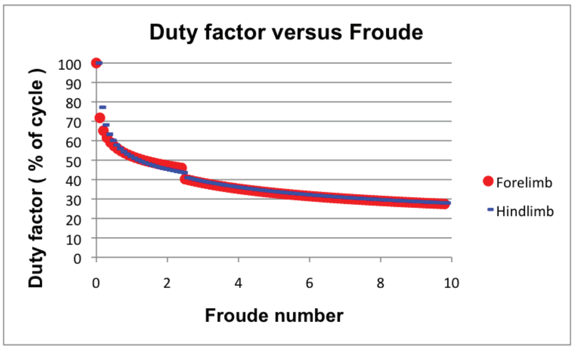

Figure 3: *Duty factors versus Froude number*

In order to predict a horse's stride length and duty factors, one obtains corresponding measurements from other cursorial quadrupeds across a large range of Froude values as found in [AJ83]. This data is then used to calculate the power law equations presented in Table 2.

Figure 3 shows a plot of the forelimb and hindlimb duty factors versus Froude number as calculated by the power law equations in Table 2. As the horse's velocity increases, ground contact decreases introducing a suspension phase.

Figure 4 displays a plot of relative stride length (the ratio of stride length to hip height) versus Froude number. To calculate the actual stride length, we use Equation 2. From this we can then calculate the stride frequency (strides per second) using Equation 3.

$$\text{stride length} = \text{relative stride length} \times \text{hip-height} \quad (2)$$

$$\text{stride frequency} = \text{velocity} \,/\, \text{stride length} \quad (3)$$

A plot of the stride frequency for a range of Froude values calculated using the hip-height of our horse model is shown in Figure 5. These stride frequency values control the rate of limb movement in the horse model.

Table 2: *Dynamic Similarity power law equations*

Prediction	Symmetrical	Asymmetrical
Fore duty factor	$y = 0.52 * x^{-0.14}$	$y = 0.52 * x^{-0.28}$
Hind duty factor	$y = 0.51 * x^{-0.18}$	$y = 0.53 * x^{-0.28}$
Relative stride len.	$y = 2.4 * x^{0.34}$	$y = 1.9 * x^{0.40}$

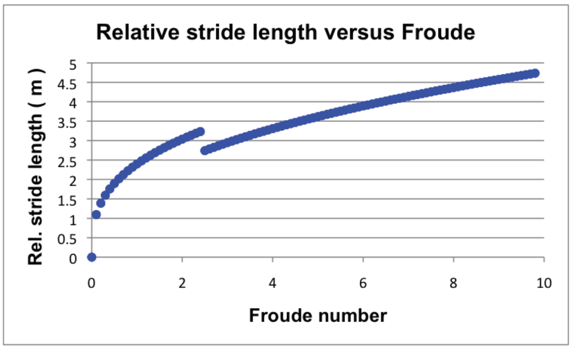

Figure 4: *Relative stride length versus Froude number.*

4. Horse model

The horse model is constructed from body-segment length data taken from [BSSB97]. Rotation values for the bones at stance are extracted from images in [Grö02]. The model's skeletal structure is displayed in Figure 6. The trunk of the model comprises of thoracic (front) and sacral (back) segments connected by the lumbosacral joint. The neck is attached to the distal end of the thoracic segment and consists of two connected bones. The head bone is connected to the neck at furthest point from the body's centre. Each of the forelimbs and hindlimbs contain 5 and 6 bones respectively. The forelimbs are positioned on either side of the thoracic segment and the hindlimbs are positioned underneath the sacral segment.

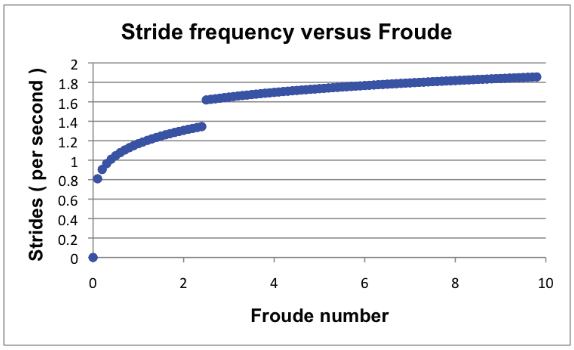

Figure 5: *Stride frequency versus Froude number (hip-height: 1.24524).*

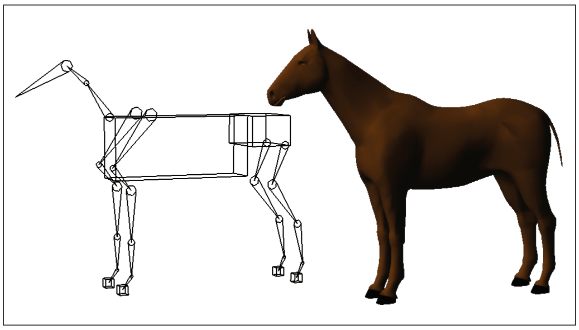

Figure 6: *Model skeletal (left) and skinned (right).*

4.1. Motion

The model's bones rotate about a single degree of freedom along the sagittal plane. Starting from the top, the rotation of each bone in a limb's hierarchy propagates downwards. The model is animated by applying a combination of rotations to the bones at each frame. The entire model is also translated a calculated distance, horizontal to the ground plane.

For a realistic animation, the model's bones must be rotated according to a specific pattern. Bone rotation data can be extracted from photographs, video or data in literature and formatted for use with the model. Rather than attempt to source, format and store separate data for every required animation, it is desirable to represent data in a simplified, intuitive manner, which allows for adjustment and reuse. To avoid storing motion data as an unintuitive set of data points, we have used both a sinusoidal and piecewise representation for cyclical and acyclical motion respectively.

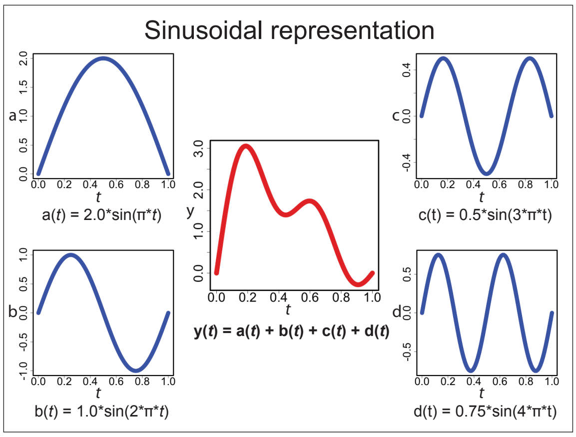

Figure 7: *Sinusoidal gait representation. The functions a(t), b(t), c(t) and d(t) are added together to give y(t), which contains the rotation data for a single bone for a full gait cycle.*

4.2. Sinusoidal gait representation

An animal's muscles tend to relax and contract in a sinusoidal manner. The rotation of a bone about a joint is often the product of multiple muscles pulling on it as they contract. In the sinusoidal gait representation, the rotation of a bone is stored as a summation of sinusoids of differing amplitude and frequency, as illustrated in Figure 7. To obtain these sinusoids, motion data is extracted from a real-life source, e.g. published joint rotation plots, and simplified by Fourier analysis. As the sinusoidal waveforms are analogous to a muscle's contribution to a bone's rotation, the representation allows for intuitive adjustment through the addition of sinusoids and performs well with optimisation techniques [MOC09]. This representation is ideal for storing repetitive, cyclical gait data however, for the acyclical gait transition data, we use a piecewise representation.

4.3. Piecewise gait representation

The piecewise representation divides a motion curve into a series of segments. Each segment has a type, a value and a position. An example piecewise representation is presented in Figure 8. In this example, the same sinusoidal curve shown in Figure 7 is represented in the piecewise form. The advantage of the piecewise representation for our transition system is that it can represent acyclical motion and be easily modified. Figure 9 shows an acyclical piecewise data representation connecting two offset cyclical waveforms.

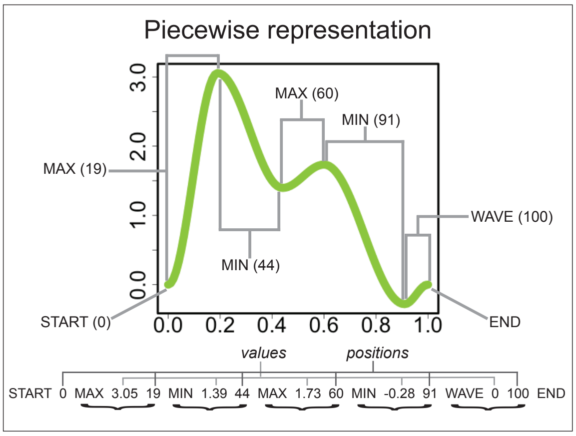

Figure 8: *A piecewise representation string is presented and plotted with annotations. This string contains rotation data for a single bone for a full transition cycle.*

5. Gait adjusters

To avoid storing separate bone rotation data for every possible Froude number an animal may use, we have developed a gait adjuster system. By exploiting the adjustability of the sinusoidal representation, adjustment data is provided for each

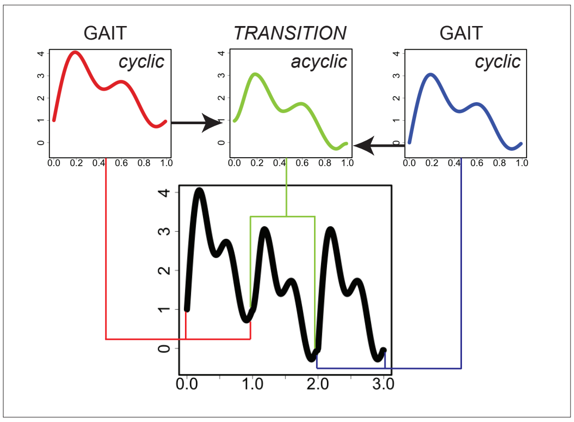

Figure 9: *Acyclical piecewise representation connects neighbouring sinusoids with differing initial offsets.*

gait. The adjustment file contains a summation of sinusoidal functions of one or more frequencies, whose amplitudes are a function of the current Froude number. A well defined gait adjuster modifies the current gait cycle to exhibit the correct limb extent and duty factor for the current Froude number. Production of a gait adjuster is complex and as such, we have developed a Curve Modifier application to aid the process.

5.1. Curve Modifier application

The Curve Modifier application allows a user to test gait adjuster data. The application displays the output animation for a single limb, alongside a visualisation of the bone rotation curves, as gait adjuster data is tested on a user-specified range of Froude numbers. During the test, duty factor and limb extent values are recorded and upon completion, a fitness score is calculated. The duty factor values are scored against Dynamic Similarity predictions whilst the limb extent measurements are scored against the expected limb extent range, from collected to extended, for the current gait. In addition to this, the graphical feedback allows a user to visually assess the gait adjuster and detect unnatural looking motion. Using this system, candidate gait adjuster data can be repeatedly modified and tested until a solution that scores well is found. This manual gait adjuster development is non-trivial however, and an automated approach is preferable.

5.2. Grammatical Evolution

We use Grammatical Evolution (GE) to evolve gait adjusters automatically using a Java-based GE system called GEVA [OHG*08]. Given a suitable grammar, GEVA produces a phenotype in the form of a gait adjuster data file. The data consists of series of summations of sinusoids of differing frequencies, whose amplitudes are a function of a free variable and may also be subject to a combination of mathematical operators and constants. This file is then passed to the Curve Modifier application for assessment.

The Curve Modifier tests the phenotype for the set range of Froude numbers. For each test, the current Froude number is substituted for the free variable. Limb extent is measured as the distance between the farthest hoof position attained in the forward and backward direction over a single cycle. A well performing gait adjuster must also ensure that any limb extensions satisfy the predicted duty factor values. The fitness values are then returned to GEVA, which evolves the next generation of phenotypes based on the received fitness scores. This process continues until a solution is found. Once gait adjuster data is prepared for each of the natural gaits, they are incorporated into the animation system.

6. Animation system

We have developed a system for animating a horse model that moves with the correct gait for a given velocity and executes gait transitions when necessary. The system requires a horse model data file as specified in Section 4, gait phase data, transition Froude values and power law equation values, as described in Section 3.1. Bone rotation data for each of the natural gaits, extracted from data plots in [BC01], is supplied in the sinusoidal representation. Finally, each of the natural gaits is given a corresponding gait adjuster file.

The system starts by reading the model file and other motion data. As the animation begins, the velocity at which the model should move is calculated based on the current Froude number and the model's hip-height. The stride frequency is calculated using Equation 3 and controls the rate at which the animation cycles through the gait data. For each change in Froude value, the gait adjusters are recalculated and added to the original gait data. This data is then converted to a set of rotation values for each bone. These values are cycled through producing the motion until a change in Froude is flagged. If this new Froude value is within the range of the current gait, the gait adjusters are recalculated to adjust the limb extent for the next cycle. If the Froude value is outside of the current gait's range, a transition is flagged.

6.1. Transitions

When a transition is flagged, the system identifies the two gaits involved in the transition. For each bone in the model, the final rotation point of the current gait cycle is stored. The gait that is being transitioned to is then created. The gait adjusters are calculated and added to the original gait data for that Froude range and the first rotation point of that gait data is stored. The transitional data is then calculated as the average of the two sinusoidal waveforms (with zero offset) it will be transitioning between. This average waveform is then converted to the piecewise representation. The start and end points of the transition data are adjusted to join up with the stored offset values, as shown previously in Figure 9.

During a transition, each limb adjusts itself to its new phase value by either increasing or decreasing its rate of movement. Gait data is spread over a number of gait cycle stages different to the standard number of stages in a cycle, chosen to be 100. To simplify the system, each limb's transition is completed over a single transition cycle in which that limb's motion is either extended, reduced or unchanged.

For each limb the phase difference value for the current gait is subtracted from the the phase difference value for the gait that is being transitioned to. If the absolute value of this number is less than 50, than the number is added to the standard number of stages in a cycle i.e. 100. If the absolute value of the number is greater than or equal to 50, the absolute value of the number is subtracted from 100 and if the original number is positive the resultant value is subtracted from 100. If the original value is negative the resultant value is added to 100. This calculated value is the number of transition cycle stages and, if the value is not equal to 100, the transition motion is spread across this extended or reduced cycle, thus changing the phase difference of the limb.

The preceding calculations ensure a transition is never excessively long or short relative to the standard number of cycle stages, creating smooth transitions between gaits.

6.2. MIDI control

Given the dynamic nature of the system, we have developed a complementary interface for it. We use a Korg nanoKONTROL slim-line MIDI controller [Kor] to adjust the model's Froude number and initiate transitions in real-time.

The animation system is controlled by five vertical sliders and a toggle button associated with each slider. An additional Transition button flags a transition (when in the manual transition mode). A simplified illustration of the controller is shown in Figure 10. The user may either manually move through gait cycles or use the automatic transition mode in which the system automatically chooses a gait to transition to based on the current Froude value.

In manual mode, each gait has a slider controlling its Froude range. To intiate a Transition the user selects a gait adjacent to the current gait, sets the new gait's slider to a desired Froude value and presses Transition. Figure 10 shows an example situation. Assuming the current gait is Walk with Froude value of 1.05, Trot is selected and its slider set to 1.8. Transition is pressed and the model transitions from Walk to Trot during the following cycle. The system allows for continuous up and down transitions between adjacent gaits.

The automatic transition mode gives a single slider control over the animal's Froude value. The user sets this slider to some value along the entire Froude range. Depending on a setup option, the application either checks for a Froude value change at the start of each cycle or upon a press of the Transition button. If the Froude value has not changed, the

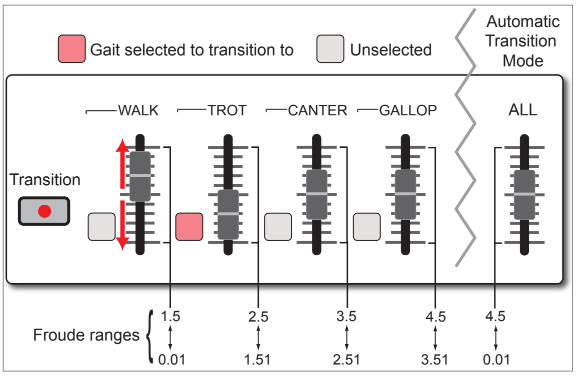

Figure 10: *Simplified illustration of the MIDI controller. A user selects a gait for transition, sets the slider to a Froude value and then presses the Transition button.*

model continues to move at it was, otherwise the application checks if the change indicates a transition. The model will then transition if required. If it is flagged as a simple change of Froude within the current gait, the gait adjusters will be calculated and applied. To simplify the system, any requests for transitions between non-adjacent gaits are ignored.

7. Results

Table 3: *Animation plan*

Gait	Froude	Cycles
Walk	1.0	3
Trot	2.0	3
Canter	3.0	3
Gallop	4.0	3
Canter	3.0	3
Trot	2.0	3
Walk	1.0	3

The system is tested on the sequence of gaits and transitions shown in Table 3. We then analyse the calculated transition phase data and the effect of the gait adjusters and calculated transitions on the bone rotation data.

7.1. Transition phase data

Figure 11 presents limb phase data from a test run. The diagrams display the phase difference between limbs (thick black bars represent ground contact) during a continuous run starting from the top-left cycle. The data is divided into standard gait cycles by the vertical bars. The diagram does not display the data temporally and does not indicate velocity.

From this diagram it can be noted that each of the gaits has the appropriate limb phase relationship as presented in

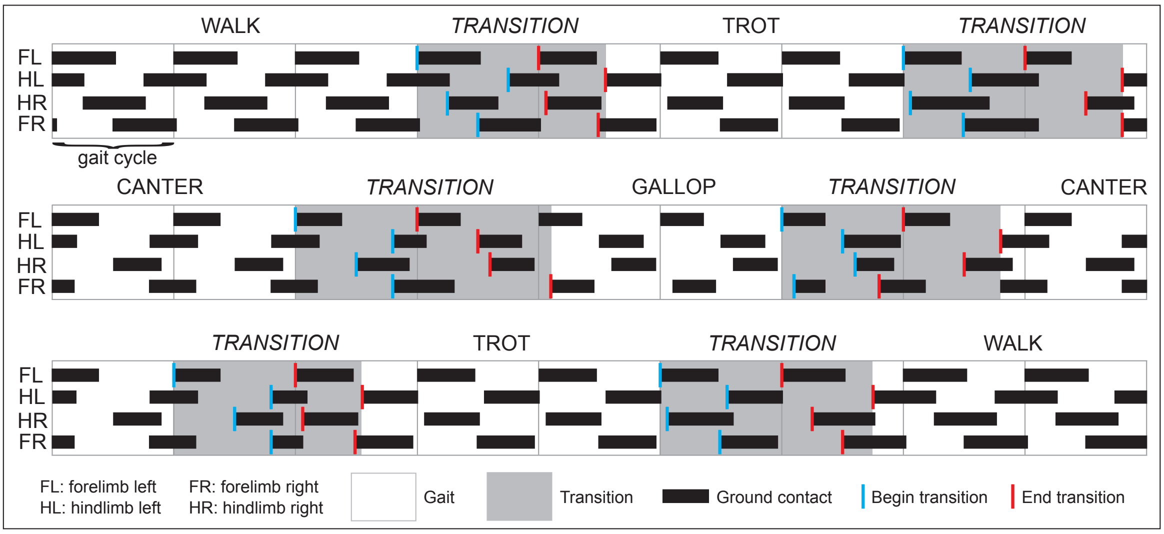

Figure 11: *Phase differences for gaits and transitions as calculated by the application.*

Figure 1. The gaits recorded through the sequence of upward transitions are identical to the gaits recorded during the downward sequence. The diagram also displays a clear difference between the symmetrical and asymmetrical gaits.

Table 4: *Recorded transitional cycle stages*

Transition	FL	HL	HR	FR
Walk to Trot	100	80	81	99
Trot to Canter	100	125	144	131
Canter to Gallop	100	70	110	130
Gallop to Canter	100	130	90	70
Canter to Trot	100	75	56	69
Trot to Walk	100	120	119	101

The transitional cycle stage values for each limb are presented in Table 4. In each transition the system has ensured that the increase or decrease in movement rate is not extreme compared to the surrounding data.

7.2. Bone rotation data

The plot of a bone's rotation during a test run is presented in Figure 12. The stretching effect of the transition cycle can be seen in the upper image as can the effect of the gait adjuster. For this example, identical gait data is supplied for the four natural gaits. The evolved gait adjusters dynamically adjust the data to provide greater limb extent in the faster gaits and this is reflected in the curve amplitudes.

The lower image displays rotations with respect to a time unit (length of a Walk gait cycle). As the model transitions up through the gaits, the frequency of the strides increases.

8. Conclusions and future work

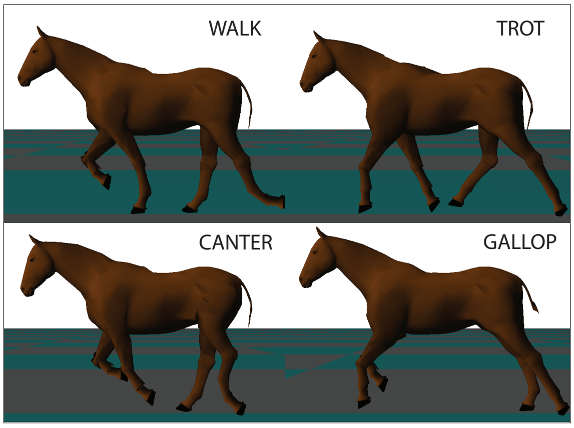

Figure 13: *The model in motion for the four natural gaits.*

We have produced a system in which a horse model moves with a gait appropriate to its velocity and transitions smoothly. The dynamic limb extent provides an extra element of realism while the MIDI control system gives the user precision control over the motion. Figure 13 shows screenshots of the four natural gaits.

In future, the system could be expanded to incorporate more natural world observations such as back flexibility and realistic looking neck and tail balancing motions. The gait library could be expanded, transitions could be spread across multiple cycles and occur between non-adjacent gaits. Overall, this approach could be applied to physics-based quadrupedal animation, developing the gait adjusters concept for the challenging issues of gait stability and balance.

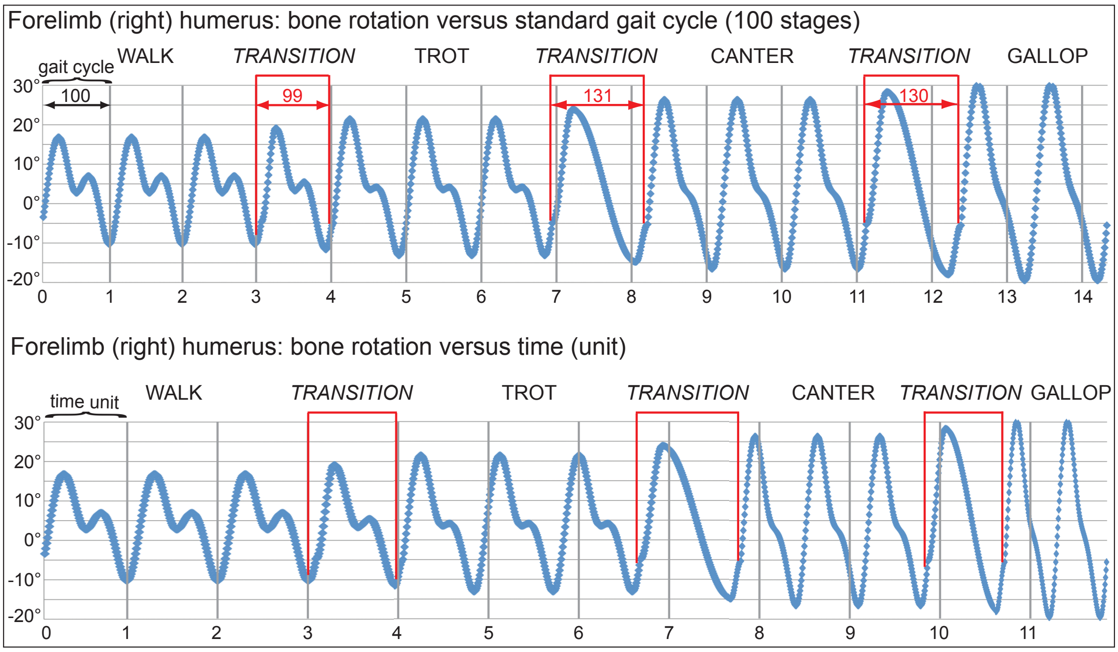

Figure 12: *Rotation of right forelimb's humerus during locomotion. The upper image plots rotation against standard gait cycle. Transition durations are given in gait cycle stages. The lower image plots rotation againt a time unit equal to one walk cycle.*

References

[AJ83] Alexander R. M., Jayes A. S.: A dynamic similarity hypothesis for the gaits of quadrupedal mammals. In *Journal of Zoology (London)*, vol. 201. The Zoological Society of London, 1983, pp. 135–152.

[BC01] Back W., Clayton H. M.: *Equine Locomotion*. Harcourt Publishers, 2001.

[BSSB97] Buchner H. H. F., Savelberg H. H. C. M., Schamhardt H. C., Barneveld A.: Inertial properties of dutch warmblood horses. In *Journal of Biomechanics*, vol. 30. Elsevier Science Ltd., 1997, pp. 653–658.

[Grö02] Grönberg P.: *ABC of the Horse: A handbook of equine anatomy, biomechanics and conditioning*. PG-Team, 2002.

[Har93] Harris S. E.: *Horse Gaits, Balance and Movement: The natural mechanics of movement common to all breeds*. Macmillan General Reference, 1993.

[HM01] Herr H. M., McMahon T. A.: A galloping horse model. In *The International Journal of Robotics Research* (2001), vol. 20, Sage Publications, pp. 26–37.

[Kor] Korg: *Korg nanoKONTROL specification*. Korg. http://www.korg.co.uk/downloads/nano/support/nanoKONTROL_OM_E2.pdf.

[LS02] Lin J.-N., Song S.-M.: Modeling gait transitions of quadrupeds and their generalization with cmac neural networks. In *IEEE Transactions on Systems, Man and Cybernetics*, vol. 32. IEEE, Aug. 2002, pp. 177–189.

[Mar74] Marey E. J.: *Animal Mechanism: A Treatise on Terrestrial and Aerial Locomotion*, 2nd ed. The International Scientific Series. Henry S. King & Co., 1874.

[MOC09] Murphy J. E., O'Neill M., Carr H.: Exploring grammatical evolution for horse gait optimisation. In *12th European Conference on Genetic Programming* (2009), Springer.

[Muy85] Muybridge E.: *Horses and other animals in motion*. Dover Publications, 1985.

[OHG*08] O'Neill M., Hemberg E., Gilligan C., Bartley E., McDermott J., Brabazon A.: *GEVA - Grammatical Evolution in Java (v1.1)*. University College Dublin, Ireland, December 2008.

[OR03] O'Neill M., Ryan C.: *Grammatical Evolution*. Kluwer Academic Publishers, 2003.

[SRH*09] Skrba L., Reveret L., Hétroy F., Cani M.-P., O'Sullivan C.: Animating quadrupeds: Methods and applications. In *Computer Graphics Forum*, vol. 28. The Eurographics Association and Blackwell Publishing Ltd., 2009, pp. 1541–1560.

[VO05] Vaughan C. L., O'Malley M. J.: Froude and the contribution of naval architecture to our understanding of bipedal locomotion. In *Gait and Posture*, vol. 21. Elsevier Science Ltd., 2005, pp. 350–362.